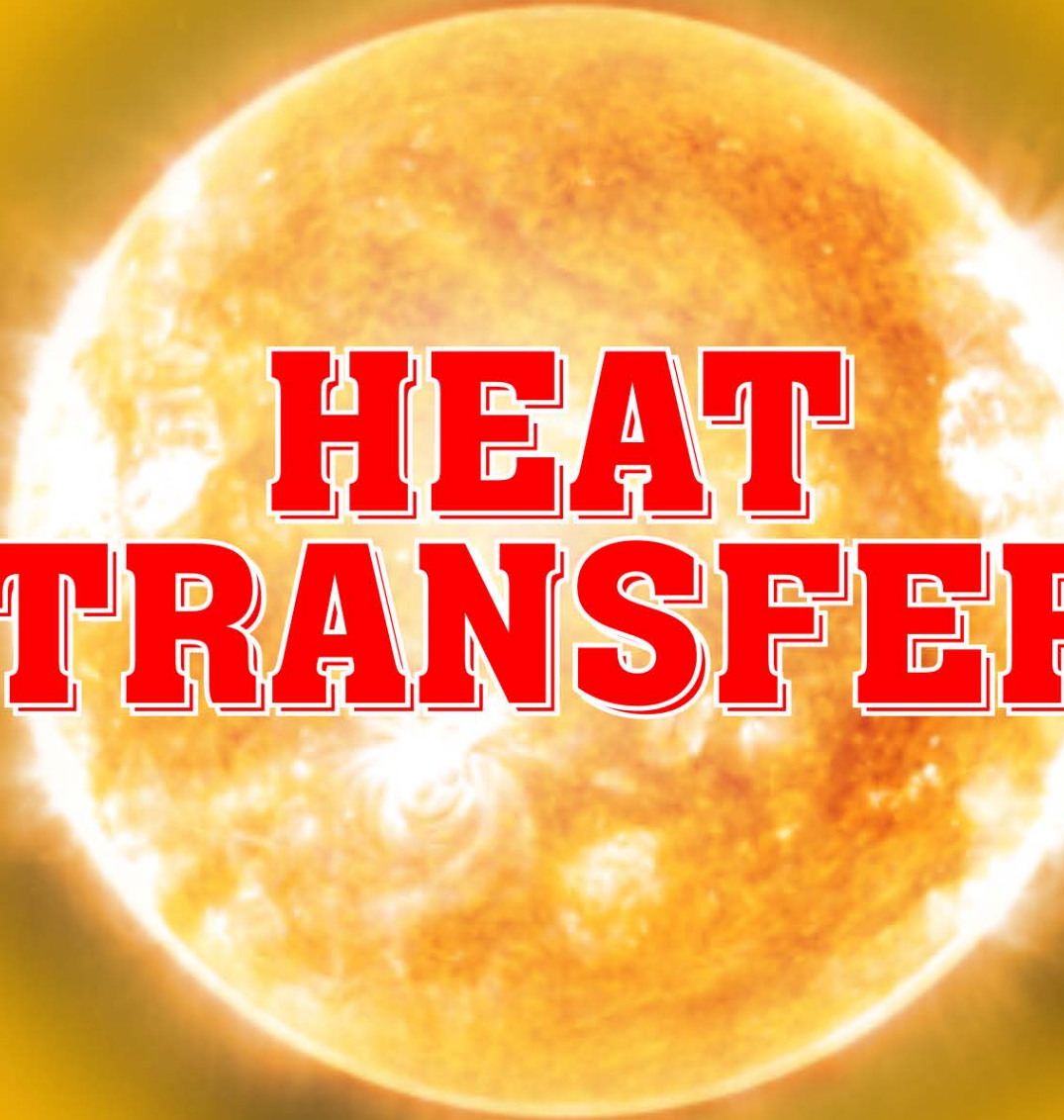


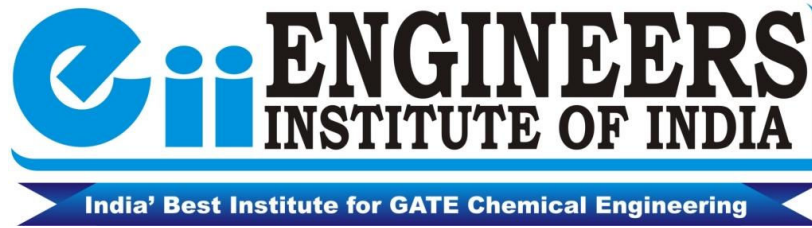


India's Best Institute for CHEMICAL ENGINEERING

CHEMICAL ENGINEERING
REVISED AS PER GATE



**HEAT
TRANSFER**



CHEMICAL ENGINEERING

Revised as Per New GATE Syllabus

STUDY MATERIAL

HEAT TRANSFER

GATE Syllabus: CHEMICAL ENGINEERING

Heat Transfer: Equation of Energy, Steady and unsteady heat conduction, convection and radiation, thermal boundary layer and heat transfer coefficients, boiling, condensation and evaporation; types of heat exchangers and evaporators and their process calculations. Design of double pipe, shell and tube heat exchangers, and single and multiple effect evaporators.

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List of Topics in GATE 2024 paper from Heat Transfer-HT:











1M (Dimensional analysis)+ 1M (Conduction through slab)+ 2M (radiation)+ 2M (heat exchanger)+ 2M (unsteady state heat conduction)

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1.
CHAPTER**INTRODUCTION**

Heat Transfer: It is the science that seeks to product the energy (Heat) transfer that may take place between material bodies as a result of temperature difference.

- It explains how and at which rate heat energy is transferred between material bodies.
- Any transport takes place due to the presence of driving force.
- In heat transfer, the driving force is thermo potential difference that is also called temperature difference.
- Heat transfer takes place by various modes or mechanisms.
- These mechanisms and the related principles enable us to calculate the rate and extent of transport of thermal energy.
- Heat transfer also explains temperature history w.r.t. time and w.r.t place or co-ordinate.

Difference between Thermodynamics and Heat Transfer:

- Thermodynamics deal with the relation between heat and other forms of energy whereas heat transfer concerned with the analysis of the rate of heat transfer.
- Thermodynamics deals with the systems at equilibrium, heat transfer deals with the system that lack of thermal equilibrium, and hence it is a non-equilibrium phenomenon.
- Thermodynamics is used to find out the amount of thermal energy is needed for changing a system from one equilibrium state to another.
- Thermodynamics doesn't tell anything about the history of the heat transfer rate or temperature while changing from one equilibrium state to another between material bodies.

Modes of Heat Transfer:

1. Conduction 2. Convection 3. Radiation

1. Conduction

- Conduction takes place in solid or stagnant liquid or gaseous medium due to existence of a temperature difference.
- It is not associated with the movement and displacement of particles of the medium from their original position.
- It is achieved by two mechanisms:
 - (i) Molecular interaction where heat transfer takes place by the kinetic motion or direct impact of molecules.
 - Molecules at a high energy level impart energy to adjacent molecules at lower energy levels.
 - Conduction energy transfer always exist so long as there is a temperature gradient in a system comprising molecules of a solid, liquid or gas.
 - (ii) By the drift of 'free' electrons as in the case of metallic solids.
 - The metallic alloys have a different concentration of free electrons and their ability to conduct heat energy is directly proportional to the concentration of free electrons in them.

- Heat energy is transferred from one molecule to the adjacent one through molecular vibrations in general solids, drift of free electrons in metals, collisions in gases and liquids etc but in all types, there is no change of original positions of the molecules.

Basic Law of Conduction = Fourier Law

Fourier Law :

- It is also called Joseph Fourier phenomical law. This law is an empiricial law based on observation.
- According to this law the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, and inversely proportional to the thickness of the layer.

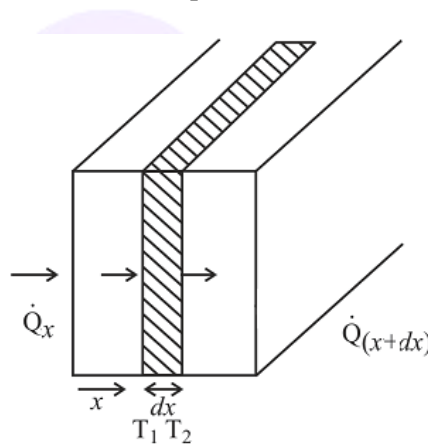
i.e.
$$\frac{\dot{Q}_x}{A} \propto \frac{\partial T}{\partial x}$$

where $\frac{\partial T}{\partial x}$ = Temperature gradient in x -direction.

$$\dot{Q}_x = -k A \frac{dT}{dx}$$

where, k = Thermal conductivity \dot{Q}_x = Heat transfer rate
 A = Area Unit of k = W/mK k depends on properties of material

- If k changes with position of material = Anisotropic material



- If k does not changes with position of material = Isotropic material.
- For simplicity of the problem, we always assume material is isotropic.
- k can be function of temperature.

$$k = k_o (1 + a T) \qquad k = k_o (1 + a T + b T^2)$$

where a and b are constants

- k_{metal} = Very high, $k_{\text{gaseous, non metal}}$ = very low.

Assumption of Fourier Law:

- The heat (energy) flow is in uni-directional.
- $\frac{dT}{dx}$ = constant and temperature profile linear.
- There is no internal heat generation.
- Material is isotropic and homogeneous.
- Conduction of heat takes place under steady state conditions.

Practice Question (Introduction)

Question-01: A steam pipe ($\epsilon = 0.5$) of 0.2m diameter has a surface temperature of 500 K and is located in large room maintained at 27°C where h convection is 20 Watt/m²K. Calculate value of overall heat transfer coefficient for convection and radiation combined.

Answer: 27.27

Heat transfer coefficient for radiation

$$hr = \sigma \epsilon (T_1^2 + T_2^2)(T_1 + T_2) \\ = 5.67 \times 10^{-8} \times 0.5(500 + 300)(500^2 + 300^2) = 7.71 \text{ watt / m}^2\text{K}$$

h overall = h convection + h radiation

$$= 20 \frac{\text{watt}}{\text{m}^2\text{K}} + 7.71 \frac{\text{Watt}}{\text{m}^2\text{K}} = 27.27 \frac{\text{Watt}}{\text{m}^2\text{K}}$$

Question-02: In a double pipe heat exchanger heat exchanger is taking place between two fluids at temperature 70°C and 50°C respectively. If thickness of inner pipe is assumed to be negligible, calculate over all heat transfer coefficient of heat exchanger, given that

$h_i = 5 \text{ watt / m}^2\text{K}$ and $h_o = 8 \text{ watt / m}^2\text{K}$.

Answer: 3.07

$$\frac{1}{V} = \frac{1}{h_i} + \frac{1}{h_o} \Rightarrow V = \frac{h_i h_o}{h_o + h_i} = \frac{40}{13} \approx 3.07 \text{ W / m}^2\text{K}.$$

Question-03: Resistance in radiation between two black bodies at temperature 400 K and 500 K respectively is equal to m²K / Watt

Answer: 0.0478

$$\sigma A(T_1^4 - T_2^4) = hr \times A(T_1 - T_2)$$

$$hr = \frac{\sigma(T_1^4 - T_2^4)}{(T_1 - T_2)}$$

$$\text{Resistance } \frac{1}{hr} = \frac{(T_1 - T_2)}{\sigma(T_1^4 - T_2^4)} = \frac{500 - 400}{5.67 \times 10^{-8} [500^4 - 400^4]}$$

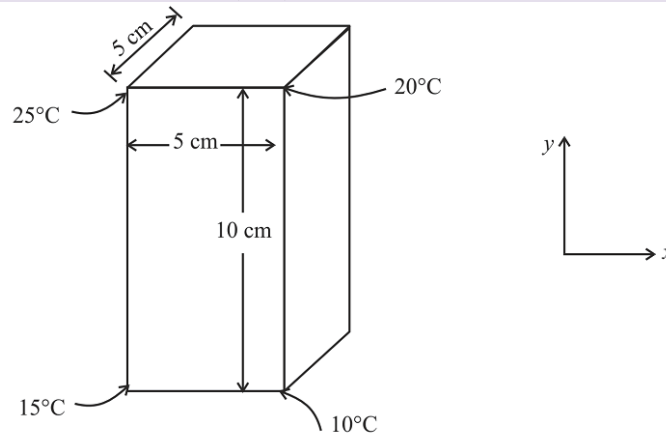
$$\frac{1}{hr} = 0.0478 \frac{\text{m}^2\text{K}}{\text{Watt}}$$

Question-04: The block of stainless steel shown below is well insulated on the front and back surfaces, and the temperature in the block varies linearly in both the x and y -directions, find:

(a) The heat flux and heat flows in the x and y -directions.

(b) The magnitude and direction of heat flux vector.

Thermal conductivity of material = 14.4 W/(mK) .



Solution:

(a) The cross-sectional areas are:

$$A_x = 10 \times 5 = 50 \text{ cm}^2 = 0.0050 \text{ m}^2 \quad A_y = 5 \times 5 = 25 \text{ cm}^2 = 0.0025 \text{ m}^2$$

$$\text{So, the heat fluxes are: } \hat{q}_x = -k \frac{\partial T}{\partial x} = -k \frac{\Delta T}{\Delta x} = -14.4 \frac{(-5)}{(0.05)} = 1440 \text{ W/m}^2$$

$$\hat{q}_y = -k \frac{\partial T}{\partial y} = -k \frac{\Delta T}{\Delta y} = -14.4 \left(\frac{10}{0.1} \right) = -1440 \text{ W/m}^2$$

$$\text{So, the heat flows are: } q_x = \hat{q}_x A_x = 1440 \times 0.005 = 7.2 \text{ W}$$

$$q_y = \hat{q}_y A_y = -1440 \times 0.0025 = -3.6 \text{ W}$$

$$(b) \quad \vec{q} = \hat{q}_x \vec{i} + \hat{q}_y \vec{j}$$

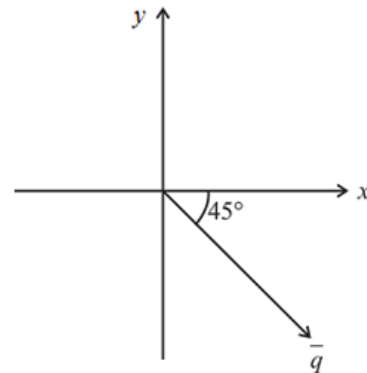
$$\vec{q} = 1440 \vec{i} - 1440 \vec{j}$$

$$|\vec{q}| = [(1440)^2 + (-1440)^2]^{0.5} = 2036.5 \text{ W/m}^2$$

The angle θ , between the heat flux vector and the x -axis is calculated as follows:

$$\tan \theta = \frac{\hat{q}_y}{\hat{q}_x} = \frac{-1440}{1440} = -1.0 \quad \theta = -45^\circ$$

So the directions of heat flow:



Question-05: The heat flux, q is 5000 W/m^2 at the surface of an electrical heater. The heater temperature is 150°C when it is cooled by air at 50°C . What is the average convective heat transfer coefficient, \bar{h} ? What will the heater temperature be if the power is reduced so that q is 3000 W/m^2 ?

Solution :
$$\bar{h} = \frac{q}{\Delta T} = \frac{5000}{(150 - 50)} = 50 \text{ W/(m}^2\text{K)}$$

If the heat flux is reduced, \bar{h} should remain unchanged during forced convection. Thus,

$$\Delta T = T_{\text{heater}} - 50^\circ \text{C} = \frac{q}{h} = \frac{3000 \text{ W/m}^2}{50 \text{ W/(m}^2\text{.K)}} = 60 \text{ K}$$

So,
$$\Delta T = T_{\text{heater}} = 60 + 50 = 110^\circ \text{C}$$

Question-06: Consider a person standing in a room temperature maintained at 22°C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 5°C in winter and 25°C in summer. Determine the rate of radiation heat transfer between the person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are 1.0 m^2 and 30°C . Assume no convective heat transfer. The emissivity of person $\epsilon = 0.95$.

Solution :
$$\begin{aligned} \dot{Q}_{\text{rad, winter}} &= \epsilon \sigma A_s (T_s^4 - T_{\text{surr, winter}}^4) \\ &= 0.95 \times \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \right) (1.0 \text{ m}^2) \times [(30 + 273)^4 - (5 + 273)^4] \text{ K}^4 = 132.3 \text{ W} \\ \dot{Q}_{\text{rad, summer}} &= \epsilon \sigma A_s (T_s^4 - T_{\text{surr, summer}}^4) \\ &= 0.95 \times \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \right) (1.0 \text{ m}^2) \times [(30 + 273)^4 - (25 + 273)^4] \text{ K}^4 = 29.23 \text{ W} \end{aligned}$$

2. CHAPTER

CONDUCTION

One dimensional heat conduction analysis:

→ Suppose the one dimensional system shown in fig. when the system is in steady state *i.e.*, the temperature does not change with time, and then we need only integrate the equation.

$$Q_{\text{conduction}} = -k A \frac{dT}{dx}$$

And substitute the appropriate values to solve for desired quantity.

→ Let the general case where the temperature may be changing or varying with time and heat sources may be present within the body.

→ For the element of thickness dx the energy balance may be made.

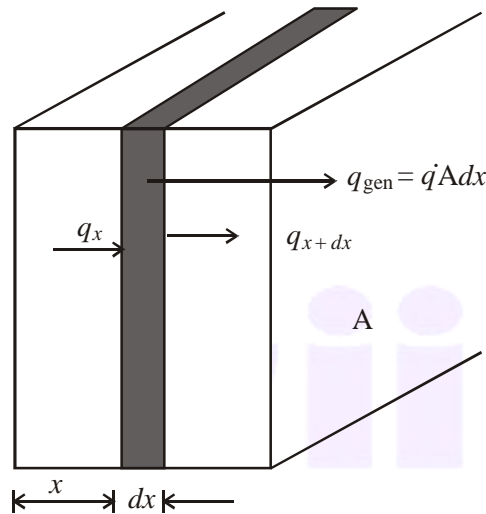


Figure: Elemental volume for one-dimensional heat conduction analysis

Energy conducted in left face + Heat generated within element = Change in internal energy + Energy conducted out right face.

$$q_x + q_{\text{gen}} = \rho C A \frac{\partial T}{\partial t} dx + q_{x+dx} \quad \dots(1)$$

As we know,

$$q_x = -k A \frac{\partial T}{\partial x}, \quad q_{\text{gen}} = \dot{q} A dx \quad \& \quad q_{x+dx} = -k A \frac{\partial T}{\partial x} \Big|_{x+dx} = -A \left[k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \right]$$

Now equation (1) becomes,

$$-k A \frac{\partial T}{\partial x} + \dot{q} A dx = -A \left[k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \right] + \rho C A \frac{\partial T}{\partial t} dx$$

where, \dot{q} = energy generated per unit volume, W/ m³

C = specific heat of material, J/kg K

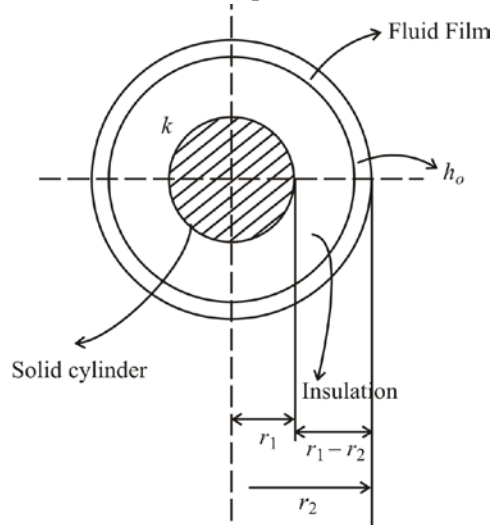
ρ = density, kg/ m³

Then finally one dimensional equation becomes

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho C \frac{\partial T}{\partial t}$$

Critical Thickness of Insulation

Critical Thickness: The thickness upto which heat flow increases and after which heat flow decreases is said to be critical thickness. For any cylindrical and spherical body, on increasing insulation thickness resistance through conduction decreases but resistance due to convection increases so the critical thickness should be optimum for heat transfer. This optimum thickness is called critical thickness.



For Cylinder: Rate of heat transfer from the surface of the cylinder to the surroundings:

$$Q = \frac{2\pi L(T_1 - T_{\text{air}})}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{h_o r_2}}$$

Q becomes maximum when denominator becomes minimum. r_1 fix and r_2 varies so :

$$\frac{d}{dr_2} \left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{k} + \frac{1}{h_o r_2} \right] = 0 \quad \Rightarrow \quad \frac{1}{k} \cdot \frac{1}{r_2} + \frac{1}{h_o} (-r_2^{-2}) = 0$$

$$\frac{1}{k} \cdot \frac{1}{r_2} + \frac{1}{h_o} \left(\frac{-1}{r_2^2} \right) = 0 \quad \Rightarrow \quad \frac{1}{k} - \frac{1}{h_o r_2} = 0 \quad (r_2) = (r_c) = \frac{k}{h_o}$$

For Sphere: Rate of Heat Transfer:

$$Q = \frac{(T_1 - T_{\text{air}})}{\left[\frac{r_2 - r_1}{4\pi k r_1 r_2} \right] + \frac{1}{4\pi r_2^2 \cdot h_o}}$$

$$\frac{d}{dr_2} \left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2} + \frac{1}{4\pi r_2^2 \cdot h_o} \right] = 0 \quad \Rightarrow \quad \frac{d}{dr_2} \left[\frac{1}{k r_1} - \frac{1}{k r_2} + \frac{1}{r_2^2 h_o} \right] = 0$$

$$\frac{1}{k r_2^2} - \frac{2}{r_2^3 h_o} = 0 \quad r_2^3 h_o = 2k r_2^2 \quad \Rightarrow \quad r_c = r_2 = \frac{2k}{h_o}$$

Practice Problem

Question-1: A Thick wall of spherical shell is made of hard rubber and is being used to store hot oil. The inner and outer radius of shell are 5mm and 20 mm respectively. Calculate thermal resistance of shell if thermal conductivity of hard rubber is $K = 0.151 \text{ W/mK}$

Answer: $R_{Th} = 284.63 \frac{\text{km}^2}{\text{W}}$

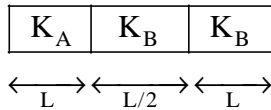
Hint: $R_{\text{Spherical shell}} = \frac{1}{4\pi K} \left[\frac{1}{r_i} - \frac{1}{r_o} \right]$

Question-2: For a given solid material thermal conductivity varies as $K = 2[1 + 0.7T]$, find average thermal conductivity across the solid material when temperature 20°C and 60°C is maintained at both side of solid in steady state condition. T is given in K.

$K_m = \dots\dots\dots$

Hint: $K_m = K_0 \left[1 + \beta \frac{(T_1 + T_2)}{2} \right] = 2 \times [1 + 0.7 \times 313] = 440.2$

Question-3: Effective thermal conductivity for given composite wall is given by:



(a) $K_{\text{eff}} = \frac{3/2}{\frac{1}{K_A} + \frac{2}{K_B} + \frac{1}{K_C}}$

(b) $K_{\text{eff}} = \frac{3}{\frac{1}{K_A} + \frac{2}{K_B} + \frac{1}{K_C}}$

(c) $K_{\text{eff}} = \frac{3}{\frac{1}{K_A} + \frac{1}{2K_B} + \frac{1}{K_C}}$

(d) $K_{\text{eff}} = \left[\frac{3/2}{\frac{1}{K_A} + \frac{1}{2K_B} + \frac{1}{K_C}} \right]$

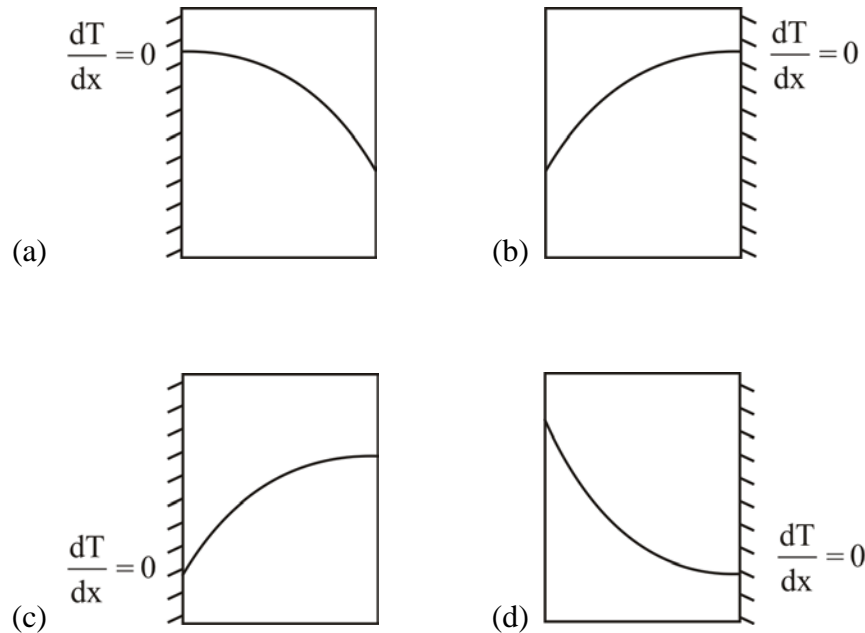
Answer: d

$$\left(\frac{3L}{2} \right) \frac{1}{K_{\text{eff}}} = \frac{L}{K_A} + \frac{L/2}{K_B} + \frac{L}{K_C}$$

$$\frac{3/2}{K_{\text{eff}}} = \frac{1}{K_A} + \frac{1}{2K_B} + \frac{1}{K_C}$$

$$K_{\text{eff}} = \left[\frac{3/2}{\frac{1}{K_A} + \frac{1}{2K_B} + \frac{1}{K_C}} \right] = \left[\frac{1.5}{\frac{1}{K_A} + \frac{1}{2K_B} + \frac{1}{K_C}} \right]$$

Question-4: Temperature profile for one dimensional steady state with heat consumption is given by



Answer: (d) $\frac{d^2T}{dx^2} - \frac{q^o}{K} = 0$

SOLVED NUMERICALS FOR PRACTICE OF GATE & PSU'S

1. A thermo-pane window consists of two sheets of glass each 6 mm thick, separated by a layer of stagnant air also 6 mm thick. Find the percentage reduction in heat loss from this plane as compared to that of a single sheet of glass 6 mm thickness. The temperature drop between inside and outside remain same at 15° C. Thermal conductivity of glass is 30 times that of air.

Solution: $\frac{q\Delta T}{A} = \frac{1}{R_{total}}$ where, $R_{total} = R_1 + R_2 + R_3$ & $R_{th} = \frac{L}{K}$

Data given, Thermal conductivity of glass = k

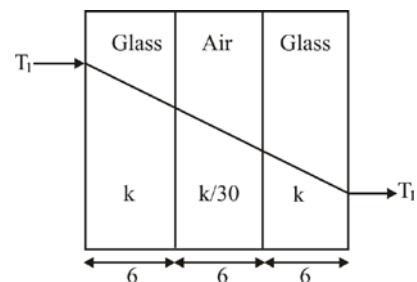
Thermal conductivity of air = $\frac{k}{30}$

We know that temperature drop

$T_1 - T_2 = \Delta T = 15^\circ C$; Let $A = 1mm^2$

$$q_1 = \frac{\Delta T}{\sum R_{th}} = \frac{15}{\frac{6}{k} + \frac{6 \times 30}{k} + \frac{6}{k}} = \frac{15}{192}k \quad ; \quad q_2 = \frac{15}{\frac{6}{k}} = \frac{15}{6}k$$

Reduction in heat loss is given by $= \frac{q_2 - q_1}{q_2} \times 100 = \frac{\left(\frac{15}{6}\right) - \left(\frac{15}{192}\right)}{\frac{15}{6}} \times 100 = 96.90\%$



3. CHAPTER

HEAT TRANSFER FROM EXTENDED SURFACE (FINS)

- The heat conducted through solids, walls has to be without stopping dissipated to the surroundings or environment to keep the system in a steady state condition.
- In engineering applications large quantities of heat have to be dissipated from small areas.
- Heat transfer by convection between the surface and the fluid surroundings can be enhanced by attaching to the surface thin strips of metals called fins.
- The fins increase the effective area of the surface; therefore, it increases the heat transfer by convection.
- Fins are also said to be 'extended surfaces'.
- Fins or 'extended surfaces' are manufactured in various geometries, depending upon the practical applications, some of which are.
 - (i) The ribs attached along the length of the tube are said to be longitudinal fins.
 - (ii) The concentric annular discs around a tube are known as circumferential fins.
 - (iii) Pin fins or spines are rods protruding from a surface.

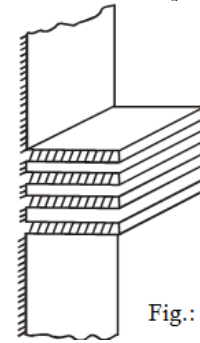


Fig.: Plate fins.

Application of fins:

- (i) Cooling of electronic components.
 - (ii) Cooling of motor cycle engines, compressors, electric motors, transformers, refrigerators, high efficiency boiler super heater tubes etc.
 - (iii) Radiator of an automobile.
- The material used for manufacturing fin must have high thermal conductivity to enhance heat transfer eg. Copper, aluminium.

Analysis of Heat flow through fins:

The following assumptions are made for the analysis of heat flow through fins:

- (i) Steady state heat conduction.
- (ii) No heat generation within the fin.
- (iii) Homogeneous and isotropic fin material.
- (iv) Uniform heat transfer coefficient (h) over the entire surface of the fin.
- (v) Negligible radiation.
- (vi) Negligible contact thermal resistance.

Heat flow through rectangular fin:

- The problem of finding of heat flow through a fin necessary the knowledge of temperature distribution through it.
- This can be achieved by regarding the fin as a metallic plate connected as its base to a heated wall and transferring heat to a fluid by convection.
- The temperature distribution in a fin will rely on the properties of both the fin material and the surrounding fluid.

Following figure shows a thin rectangular fin of uniform cross-section. The major heat conduction is along the x -axis whereas the convection loss takes place from the upper and under surfaces of the fin.

SOLVED NUMERICALS FOR PRACTICE OF GATE & PSU'S

- (1.) One end of a very long aluminium rod is connected to a wall at 140°C , while the other end protrudes into a room whose air temperature is 15°C . The rod is 3 mm in diameter and the heat transfer coefficient between the rod surface and environment is $300 \text{ W}/(\text{m}^2\cdot\text{K})$. Estimate the total heat dissipated by the rod taking its thermal conductivity as $150 \text{ W}/(\text{m}\cdot\text{K})$.

Solution:

$$a = \sqrt{hPkA(T_o - T_{\infty})}$$

$$k = 150 \text{ W}/\text{mk}, P = \pi d = 9.424 \times 10^{-3} \quad A = \frac{\pi}{4} d^4 = 7.068 \times 10^{-6} \text{ m}^2$$

$$T_o = 140^{\circ}, \quad T_{\infty} = 15^{\circ} \text{ C}$$

$$Q = \sqrt{(300 \times 9.424 \times 10^{-3}) \times 150 \times (7.068 \times 10^{-6})} (140 - 15)$$

$$Q = 0.0547 \times 125 \quad \boxed{Q = 6.843 \text{ W}}$$

- (2.) A turbine blade 6cm long and having a cross sectional area 4.65 cm^2 and perimeter 12cm, is made of steel ($k = 23 \text{ W}/\text{m}\cdot\text{K}$). The temperature at the root is 500°C . The blade is exposed to a hot gas at 870°C . The heat transfer coefficient between the blade surface and gas is $442 \text{ W}/(\text{m}^2\cdot\text{K})$. Find the distribution and rate of heat flow at the root of the blade. Consider the tip of the blade to be insulated.

Solution:

$$\frac{T - T_{\infty}}{T_o - T_{\infty}} = \frac{\cos h[m(L - x)]}{\cosh h(mL)}$$

$$L = 6 \times 10^{-2} \text{ m}, A = 4.65 \times 10^{-4}, P = 12 \times 10^{-2}$$

$$k = 23 \text{ W}/\text{mk}, h = 442 \text{ W}/\text{m}^2\text{k} \quad T_o = 500^{\circ} \text{ C}, T_{\infty} = 870^{\circ} \text{ C}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{442 \times (12 \times 10^{-2})}{(4.65 \times 10^{-4})(23.3)}} \Rightarrow m = 69.96$$

$$T - T_{\infty} = -370 \frac{\cos h[69.96(6 \times 10^{-2} - x)]}{\cosh h[69.96(6 \times 10^{-2})]}$$

$$Q = \sqrt{hPkA} \theta \tan h(mL) = \sqrt{442 \times 12 \times 10^{-2} \times 23.3 \times 4.65 \times 10^{-4}} (-370) \tan h(69.96 \times 6 \times 10^{-2})$$

$$= 0.758 \times (-370)(0.9995) \Rightarrow \boxed{Q = -286.3 \text{ W}}$$

- (3.) A finned surface consists of root or base area of 1 m^2 and fin surface area of 2 m^2 . The average heat transfer coefficient for finned surface is $20 \text{ W}/(\text{m}^2\cdot\text{K})$, effectiveness of fins provided is 0.75. When finned surface with root or base temperature of 50°C is transferring heat to a fluid at 30°C , then calculate the rate of heat transfer:

Solution:

$$Q_{fin} = (\sqrt{hPkA_c}) \theta_0 = \sqrt{20 \times 1 \times 20 \times 1} \times 0.75 \times 20 = 300 \text{ W} \quad \left[\varepsilon_{fin} = \sqrt{\frac{kP}{hA_c}} \right]$$

$$\varepsilon = \frac{Q_{fin}}{Q_{without \text{ fin}}} \Rightarrow Q_{without \text{ fin}} = \frac{300}{0.75} = 400 \text{ W} \quad \text{Note: If } \varepsilon_{fin} < 1; \text{ fins behave like insulator}$$

Question: A cylindrical fin of diameter 24 mm is attached horizontally to a vertical planar wall. The heat transfer rate from the fin to surrounding air is 60% of the heat transfer rates. If the entire fin were at the wall temperature. If the fin effectiveness is 10, its length is _____ mm. **(GATE-2022)**

Answer: 94

$$\eta = \frac{Q_{\text{actual}}}{Q_{\text{entire fin at base temperature}}} = 0.6$$

$$\frac{\epsilon}{\eta} = \frac{A_S(\text{surface area of fin})}{A_C(\text{cross section of fin})} = \frac{10}{0.6} = \frac{\pi D l + \frac{\pi}{4} D^2}{\frac{\pi}{4} D^2}$$

$$\frac{100}{6} = \frac{\pi \times 0.024 l + \frac{\pi}{4} (0.024)^2}{\pi (0.024)^2} \Rightarrow l = 94 \text{ mm}$$





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